

# A new CPT-even and Lorentz-Violating nonminimal coupling in the Dirac equation

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In this work, we propose a CPT-even and Lorentz-violating dimension-five nonminimal coupling between fermionic and gauge fields, involving the CTP-even and Lorentz-violating gauge tensor of the SME. This nonminimal coupling modifies the Dirac equation, whose nonrelativistic regime is governed by a Hamiltonian which induces new effects, such as an electric-Zeeman-like spectrum splitting and an anomalous-like contribution to the electron magnetic moment, between others. Some of these new effects allows to constrain the magnitude of this nonminimal coupling in 1 part in  $10^{16}$ .

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## I. INTRODUCTION

The Standard Model Extension (SME) [1, 2] has been the usual framework for investigating signals of Lorentz violation in physical systems. The SME is the natural framework for studying properties of physical systems with Lorentz-violation since it includes Lorentz-violating terms in all sectors of the minimal standard model. The Lorentz-violating (LV) terms are generated as vacuum expectation values of tensors defined in a high energy scale. The SME is a theoretical framework which has inspired a great deal of investigations in recent years. Such works encompass several distinct aspects involving: fermion systems and radiative corrections [3, 4], CPT- probing experiments [5], the electromagnetic CPT- and Lorentz-odd term [6–8], the nineteen electromagnetic CPT-even and Lorentz-odd coefficients [9–11]. Recently, some studies involving higher dimensional operators have also been reported with great interest [12–14]. These many contributions have elucidated the effects induced by Lorentz violation and served to set up stringent upper bounds on the LV coefficients.

Some time ago, a Lorentz-violating and CPT-odd non-minimal coupling between fermions and the gauge field was proposed [15] at the form

$$D_\mu = \partial_\mu + ieA_\mu + i\frac{g}{2}\epsilon_{\mu\lambda\alpha\beta}(k_{AF})^\lambda F^{\alpha\beta}, \quad (1)$$

in the context of the Dirac equation,  $(i\gamma^\mu D_\mu - m)\Psi = 0$ . Here, the fermion spinor is  $\Psi$ , while  $(k_{AF})^\mu = (v_0, \mathbf{v})$  is the Carroll-Field-Jackiw four-vector, and  $g$  is the constant that measures the nonminimal coupling magnitude. The analysis of the nonrelativistic limit revealed that this coupling provides a magnetic moment ( $g\mathbf{v}$ ) for uncharged particles [15], yielding an Aharonov-Casher phase for its wavefunction. It was also shown that this particular non-minimal coupling induces topological phases in more general contexts [16]. Its effects on the Hydrogen spectrum were studied in Ref. [17], while its influence in the dynamics of the Aharonov-Bohm-Casher problem was analyzed in Ref. [18]. Recently, this coupling was considered

in the context of fermion-fermion ultrarelativistic scattering, with the evaluation of corrections to the corresponding cross section and establishment of an upper bound as tight as 1 part in  $10^{12}$  [19]. Another recent work in this context was developed involving aspects related to the Hall effect and Landau levels [20]. Lastly, generalized versions of nonminimal couplings have been proposed to examine the induction of several types of topological and geometrical phases [21].

In the present work, we propose a new CPT-even, dimension-five, nonminimal coupling linking the fermionic and gauge fields in the context of the Dirac equation. By considering the nonrelativistic limit of the modified Dirac's equation, we explicitly evaluate the new contributions to the nonrelativistic Hamiltonian. These new terms imply a direct correction on the anomalous magnetic moment, a kind of electrical Zeeman-like effect on the atomic spectrum, and a Rashba-like coupling term. These effects are then used to impose upper bounds on the magnitude of the nonminimally coupled LV coefficients at the level of 1 part in  $10^{16}$ .

## II. A CPT-EVEN LORENTZ-VIOLATING NONMINIMAL COUPLING

We consider a nonminimal coupling involving fundamental Dirac fermions and the electromagnetic field in the context of the Dirac equation,

$$(i\gamma^\mu D_\mu - m_e)\Psi^{(e)} = 0, \quad (2)$$

where  $\Psi^{(e)}$  is the electron spinor wave function, and the covariant derivative with nonminimal coupling is

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda^{(e)}}{2}(K_F)_{\mu\nu\alpha\beta}\gamma^\nu F^{\alpha\beta}, \quad (3)$$

with  $(K_F)_{\mu\nu\alpha\beta}$  being the tensor ruling the Lorentz violation in the CPT-even electrodynamics of the SME, while  $\lambda^{(e)}$  is the electron nonminimal coupling constant. This tensor possess 19 components, whose properties and

effects have been examined since 2002 [9]. The background tensor  $(K_F)_{\alpha\nu\rho\varphi}$  has the same symmetries of the Riemann tensor,  $(K_F)_{\alpha\nu\rho\varphi} = -(K_F)_{\nu\alpha\rho\varphi}$ ,  $(K_F)_{\alpha\nu\rho\varphi} = -(K_F)_{\alpha\nu\varphi\rho}$ ,  $(K_F)_{\alpha\nu\rho\varphi} = (K_F)_{\rho\varphi\alpha\nu}$ , and a double null trace,  $(K_F)^{\alpha\beta}_{\alpha\beta} = 0$ , implying 19 components. This tensor  $(K_F)_{\alpha\nu\rho\varphi}$  can be written in terms of four  $3 \times 3$  matrices  $\kappa_{DE}, \kappa_{DB}, \kappa_{HE}, \kappa_{HB}$ , defined in Refs. [9] as:

$$(\kappa_{DE})_{jk} = -2(K_F)_{0j0k}, \quad (4)$$

$$(\kappa_{HB})_{jk} = \frac{1}{2}\epsilon_{jpq}\epsilon_{klm}(K_F)_{pqlm}, \quad (5)$$

$$(\kappa_{DB})_{jk} = -(\kappa_{HE})_{kj} = \epsilon_{kpq}(K_F)_{0jpp}. \quad (6)$$

The symmetric matrices  $\kappa_{DE}, \kappa_{HB}$  contain the parity-even components and possess together eleven independent components, while  $\kappa_{DB}, \kappa_{HE}$  possess no symmetry, having together eight components, representing the parity-odd sector of the tensor  $(K_F)$ .

The Dirac equation (2) can be explicitly written as

$$\left[ i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu + \frac{\lambda^{(e)}}{2}(K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta} - m_e \right] \Psi^{(e)} = 0, \quad (7)$$

and

$$\sigma^{\mu\nu} = \frac{i}{2}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) = \frac{i}{2}[\gamma^\mu, \gamma^\nu]. \quad (8)$$

Thus, the relevant electron Lagrangian is

$$\mathcal{L}_{(e)} = \bar{\Psi}^{(e)}(i\partial\!\!\!/ - eA - m_e + \frac{\lambda^{(e)}}{2}(K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta})\Psi^{(e)}. \quad (9)$$

Using the parametrization (4)-(6), we obtain:

$$\begin{aligned} (K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta} &= 2\sigma^{0i} \left[ (\kappa_{DE})_{ij} E^j + (\kappa_{DB})_{ij} B^j \right] \\ &+ \epsilon_{kij} \sigma^{ij} \left[ (\kappa_{HE})_{kq} E^q + (\kappa_{HB})_{kq} B^q \right], \end{aligned} \quad (10)$$

where we have used  $F_{0j} = E^j$ ,  $F_{mn} = \epsilon_{mnp} B^p$ , and  $\sigma^{0i}$ ,  $\sigma^{ij}$  are the components of the operator (8),

$$\sigma^{0i} = i \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \sigma^{ij} = - \begin{pmatrix} \epsilon_{ijk} \sigma^k & 0 \\ 0 & \epsilon_{ijk} \sigma^k \end{pmatrix}. \quad (11)$$

Note that these components are also expressed as  $\sigma^{0j} = i\alpha^j$ ,  $\sigma^{ij} = -\epsilon_{ijk}\Sigma^k$ . These results are explicitly evaluated in the following representation of the  $\gamma$ -matrices:

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (12)$$

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \quad \Sigma^k = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}.$$

with  $\sigma = (\sigma_x, \sigma_y, \sigma_z)$  being the Pauli matrices. With this notation, Eq. (10) is written as

$$(K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta} = 2i\alpha^j (\mathbb{E}^j + \mathbb{B}^j) - 2\Sigma^j (\tilde{\mathbb{E}}^j + \tilde{\mathbb{B}}^j). \quad (13)$$

where we have introduced the following definitions:

$$\mathbb{E}^k = (\kappa_{DE})_{kj} E^j, \quad \mathbb{B}^k = (\kappa_{DB})_{kj} B^j, \quad (14)$$

$$\tilde{\mathbb{E}}^k = (\kappa_{HE})_{kq} E^q, \quad \tilde{\mathbb{B}}^k = (\kappa_{HB})_{kp} B^p, \quad (15)$$

and the relation (6) was used. In the momentum coordinates,  $i\partial_\mu \rightarrow p_\mu$ , the corresponding Dirac equation is

$$\begin{aligned} i\partial_t \Psi^{(e)} &= [\alpha \cdot (\mathbf{p} - e\mathbf{A}) + eA_0 + m_e \gamma^0 \\ &- \lambda^{(e)} i\gamma^j (\mathbb{E}^j + \mathbb{B}^j) + \lambda^{(e)} \gamma^0 \Sigma^k (\tilde{\mathbb{E}}^k + \tilde{\mathbb{B}}^k)] \Psi^{(e)}. \end{aligned} \quad (16)$$

### III. NONRELATIVISTIC LIMIT

In order to investigate the role played by this nonminimal coupling, we should evaluate the nonrelativistic limit of the Dirac equation. Writing the spinor  $\Psi$  in terms of small ( $\chi$ ) and large ( $\phi$ ) two-spinors,

$$\Psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad (17)$$

the Dirac equation (16) leads to two two-component equations,

$$[E - eA_0 - m_e + LV1] \phi - [\boldsymbol{\sigma} \cdot (\mathbf{p} - e\mathbf{A}) - LV2] \chi = 0, \quad (18)$$

$$[\boldsymbol{\sigma} \cdot (\mathbf{p} - e\mathbf{A}) + LV2] \phi - [E - eA_0 + m_e - LV1] \chi = 0. \quad (19)$$

with

$$LV1 = -\lambda^{(e)} \sigma^k (\tilde{\mathbb{E}}^k + \tilde{\mathbb{B}}^k), \quad (20)$$

$$LV2 = i\lambda^{(e)} \sigma^j (\mathbb{E}^j + \mathbb{B}^j). \quad (21)$$

In this point we notice that the canonical momentum remains defined as

$$\pi = (\mathbf{p} - e\mathbf{A}), \quad (22)$$

once the term  $LV2$  appears with the same sign in Eqs. (18, 19). The small component is given by

$$\chi \simeq \frac{1}{[2m_e]} [\boldsymbol{\sigma} \cdot \pi + LV2] \phi, \quad (23)$$

which replaced in Eq. (18) leads to

$$\begin{aligned} [E - eA_0 - m_e + LV1] \phi &= \\ &= \frac{1}{[2m_e]} [(\boldsymbol{\sigma} \cdot \pi) - LV2] [(\boldsymbol{\sigma} \cdot \pi) + LV2] \phi. \end{aligned} \quad (24)$$

At first order in the Lorentz violating parameters, the following Hamiltonian is achieved:

$$\begin{aligned}
H^{(e)} = & \frac{1}{2m_e} \left[ (\mathbf{p} - e\mathbf{A})^2 - e(\boldsymbol{\sigma} \cdot \mathbf{B}) \right] + eA_0 + \lambda^{(e)} \boldsymbol{\sigma} \cdot (\tilde{\mathbb{E}} + \tilde{\mathbb{B}}) \\
& - \frac{\lambda^{(e)}}{m_e} (\mathbb{E} + \mathbb{B}) \cdot (\boldsymbol{\sigma} \times \mathbf{p}) + \frac{e\lambda^{(e)}}{m_e} (\mathbb{E} + \mathbb{B}) \cdot (\boldsymbol{\sigma} \times \mathbf{A}) \\
& + \frac{\lambda^{(e)}}{2m_e} [\partial_i \mathbb{E}^i + \partial_i \mathbb{B}^i + i\boldsymbol{\sigma} \cdot (\nabla \times \mathbb{E}) + i\boldsymbol{\sigma} \cdot (\nabla \times \mathbb{B})].
\end{aligned} \tag{25}$$

In the case we deal with uniform fields, the Hamiltonian becomes

$$\begin{aligned}
H^{(e)} = & \frac{1}{2m_e} \left[ (\mathbf{p} - e\mathbf{A})^2 - e(\boldsymbol{\sigma} \cdot \mathbf{B}) \right] + eA_0 + \lambda^{(e)} \boldsymbol{\sigma} \cdot (\tilde{\mathbb{E}} + \tilde{\mathbb{B}}) \\
& - \frac{\lambda^{(e)}}{m_e} (\mathbb{E} + \mathbb{B}) \cdot (\boldsymbol{\sigma} \times \mathbf{p}) + \frac{e\lambda^{(e)}}{m_e} (\mathbb{E} + \mathbb{B}) \cdot (\boldsymbol{\sigma} \times \mathbf{A}).
\end{aligned} \tag{26}$$

This Hamiltonian induces new effects. Note that the term  $\mathbb{E} \cdot (\boldsymbol{\sigma} \times \mathbf{p})$  is a generalization of the Rashba coupling term,  $\mathbb{E} \cdot (\boldsymbol{\sigma} \times \mathbf{p})$ , while  $\lambda^{(e)} (\boldsymbol{\sigma} \cdot \tilde{\mathbb{B}})$  implies a straightforward tree-level contribution to the anomalous magnetic moment of the electron. As another example, the term  $(\boldsymbol{\sigma} \cdot \tilde{\mathbb{E}})$  leads to a kind of electric Zeeman effect, in the total absence of magnetic field.

#### IV. NONRELATIVISTIC PHYSICAL EFFECTS

In this section, we analyze some physical effects induced by the correction terms enclosed in Eq. (26). In this sense, we particularize this Hamiltonian for some specific configurations of electric and magnetic fields.

We begin discussing the correction induced on the atomic spectrum of Hydrogen. In order to carry out the contribution associated with the term  $\boldsymbol{\sigma} \cdot \tilde{\mathbb{E}}$  involving the spin operator, it is necessary to work with the wave functions  $\Psi_{nljm_j m_s}^{(e)} = \psi_{nljm_j}(r, \theta, \phi) \chi_{sm_s}$ , suitable to treat the situations where there occurs addition of angular momenta ( $J = L + S$ ), with  $n, l, j, m_j$  being the associated quantum numbers. In this case, the correction energy is given by:

$$\Delta_E = \lambda^{(e)} \langle nljm_j m_s | \boldsymbol{\sigma} \cdot \tilde{\mathbb{E}} | nljm_j m_s \rangle. \tag{27}$$

Now, we adopt a polarized spin configuration,  $\boldsymbol{\sigma} = \sigma_z \hat{z}$ , such that

$$\boldsymbol{\sigma} \cdot \tilde{\mathbb{E}} = (\kappa_{HE})_{3j} \sigma_z E_j, \tag{28}$$

with  $E_j$  being one of the components of the electric field, and  $(\kappa_{HE})_{3j}$  a non null element of the matrix  $(\kappa_{HE})$ . Thus,

$$\Delta_E = \lambda^{(e)} (\kappa_{HE})_{3j} E_j \langle nljm_j m_s | \sigma_z | nljm_j m_s \rangle. \tag{29}$$

To complete this calculation, it is necessary to write the  $|jm_j\rangle$  kets in terms of the spin eigenstates  $|mm_s\rangle$ , which is done by means of the general expression:

$$|jm_j\rangle = \sum_{m, m_s} \langle mm_s | jm_j \rangle |mm_s\rangle, \tag{30}$$

where  $\langle mm_s | jm_j \rangle$  are the Clebsch–Gordan coefficients. Evaluating such coefficients for the case  $j = l + 1/2$ ,  $m_j = m + 1/2$ , one has  $|jm_j\rangle = \alpha_1 |m \uparrow\rangle + \alpha_2 |m + 1 \downarrow\rangle$ ; on the other hand, for  $j = l - 1/2$ ,  $m_j = m + 1/2$ , one obtains  $|jm_j\rangle = \alpha_2 |m \uparrow\rangle - \alpha_1 |m + 1 \downarrow\rangle$ , with  $\alpha_1 = \sqrt{(l + m + 1)/(2l + 1)}$ ,  $\alpha_2 = \sqrt{(l - m)/(2l + 1)}$ . Taking now into account the orthonormalization relation  $\langle m' m'_s | mm_s \rangle = \delta_{m' m} \delta_{m'_s m_s}$ , it is possible to show that Eq. (29) leads to:

$$\Delta_E = \lambda^{(e)} (\kappa_{HE})_{3j} E_j \langle jm_j | \sigma_z | jm_j \rangle, \tag{31}$$

$$\Delta_E = \pm \lambda^{(e)} (\kappa_{HE})_{3j} E_j \frac{m_j}{2l + 1}, \tag{32}$$

where the positive and negative signs correspond to  $j = l + 1/2$  and  $j = l - 1/2$ , respectively. It was also used  $\langle nljm_j m_s | \sigma_z | nljm_j m_s \rangle = m_j \hbar / (2l + 1)$ ,  $\langle nljm_j m_s | \sigma_x | nljm_j m_s \rangle = \langle nljm_j m_s | \sigma_y | nljm_j m_s \rangle = 0$ . The dependence on  $m_j$  leads to a spectrum splitting in  $(2j + 1)$  lines, representing an electric Zeeman-like effect (due to the presence of an electric field, that can be external or the atomic one). Regarding the possibility of measuring spectrum shifts as small as  $10^{-10} \text{ eV}$ , and working with a typical atomic electric field for the Hydrogen fundamental level ( $a_0 \simeq 0.529 \text{ \AA}$ ), whose magnitude is  $E \simeq 5.1 \times 10^{11} \text{ N/C} \simeq 1.2 \times 10^6 (\text{eV})^2$ , the Zeeman-like splitting of Eq. (32) will be undetectable if

$$|\lambda^{(e)} (\kappa_{HE})_{3j}| E_j < 10^{-10} (\text{eV}). \tag{33}$$

It leads to the following upper bound:

$$|\lambda^{(e)} (\kappa_{HE})_{3j}| < 8 \times 10^{-17} (\text{eV})^{-1}. \tag{34}$$

Now, an observation is worthwhile. In the derivation of the result (32) one has used the Hamiltonian (26), particularized for uniform fields. The nucleus Coulombian field, however, is not constant, opening the possibility of achieving new spectrum shifts stemming from the varying electric field terms of Eq. (25), namely,  $\nabla \cdot \mathbb{E}$ ,  $\boldsymbol{\sigma} \cdot (\nabla \times \mathbb{E})$ . Knowing the definition (14), and the Coulombian field,  $E^j = er^j / r^3$ , we obtain

$$\nabla \cdot \mathbb{E} = e/r^3 \left[ (\kappa_{DE})_{ii} - 3 (\kappa_{DE})_{ij} (\cos \theta_i) (\cos \theta_j) \right], \tag{35}$$

$$\nabla \times \mathbb{E} = -3e/r^3 \left[ \epsilon_{ijk} (\kappa_{DE})_{kp} (\cos \theta_j) (\cos \theta_p) \right], \tag{36}$$

where  $\cos \theta_i = r^i / r$ . We thus note that the expectation value of  $\nabla \cdot \mathbb{E}$  only will receive contributions from

the trace and diagonal elements of the symmetric matrix  $(\kappa_{DE})$ , while the expectation value of  $\nabla \times \mathbb{E}$  is affected only by the nondiagonal terms. In other words, the spectrum corrections given by  $\int \Psi_{nlm}^* (\nabla \cdot \mathbb{E}) \Psi_{nlm} d^3r$  and  $\int \Psi_{nlm}^* (\nabla \times \mathbb{E}) \Psi_{nlm} d^3r$ , are in general nonnull, with their values being proportional to  $\overline{(1/r^3)} = \langle nlm | 1/r^3 | nlm \rangle = [a_0^3 n^3 l(l+1/2)(l+1)]^{-1}$ . Thus, the energy shifts go as  $e \left| \lambda^{(e)} (\kappa_{DE})_{ij} \right| / (ma_0^3) \sim 9 \times 10^3 \left| \lambda^{(e)} (\kappa_{DE})_{ij} \right| (eV)^2$ , with  $e = \sqrt{1/137}$ , leading to the following upper bound,  $\left| \lambda^{(e)} (\kappa_{DE})_{ij} \right| < 1.1 \times 10^{-14} (eV)^{-1}$ , less restrictive than the one of Eq. (34), however.

Additional Hydrogen spectrum corrections could arise when one considers a nonminimal coupling for protons similar to the one here devised for electrons. This can be done proposing (for protons),  $D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda^{(p)}}{2} (K_F)_{\mu\nu\alpha\beta} \gamma^\nu F^{\alpha\beta}$ , where  $\lambda^{(p)}$  concerns the proton electromagnetic nonminimal interaction. This proposal is analogue to the one of Refs. [5]. Note that  $\lambda^{(p)} \neq \lambda^{(e)}$ , where  $\lambda^{(e)}$  is related to electron electromagnetic interaction (it is the constant that appears in Eq. (3)). In this case, the modified Dirac equation is,

$$\left[ i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu + \frac{\lambda^{(p)}}{2} (K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta} - M_p \right] \Psi^{(p)} = 0, \quad (37)$$

where  $\Psi^{(p)}$  is the proton spinor wave function and  $M_p$  is the proton mass. We now consider a scenario in which one supposes simultaneously two nonminimal coupling terms (for the electron and proton interactions). The full fermion Lagrangian is  $\mathcal{L} = \mathcal{L}_{(e)} + \mathcal{L}_{(p)}$ , where

$$\mathcal{L}_{(p)} = \bar{\Psi}^{(p)} (i\partial\!\!\!/ + e\mathcal{A} - M_p + \frac{\lambda^{(p)}}{2} (K_F)_{\mu\nu\alpha\beta} \sigma^{\mu\nu} F^{\alpha\beta}) \Psi^{(p)}, \quad (38)$$

is the proton Lagrangian and  $L_{(e)}$  is given by Eq. (9). Working out the nonrelativistic limit, one obtains a full Hamiltonian given as  $H = H^{(e)} + H^{(p)}$ , where  $H^{(e)}$  is the one of Eq. (25), and  $H^{(p)}$  is the analogue to this one for the proton, with  $m_e$  replaced by  $M_p$  and  $-e \rightarrow e$ . It contains new tree-level contributions in  $\lambda^{(p)}$  that yield spectrum corrections. Noticing that  $M_p \simeq 1836m_e$ , the terms of the proton nonrelativistic Hamiltonian proportional to  $M_p^{-1}$  will yield bounds less restrictive than the ones stemming from the electron Hamiltonian at least by a factor  $10^3$ . The unique term of  $H^{(p)}$  able to lead to a competitive bound is  $\lambda^{(p)} \sigma \cdot (\tilde{\mathbb{E}} + \tilde{\mathbb{B}})$ , implying the same bound of Eq. (34), that is,  $\left| \lambda^{(p)} (\kappa_{HE})_{3j} \right| < 8 \times 10^{-17} (eV)^{-1}$ . A detailed analysis about the spectrum corrections induced by the Hamiltonian  $H = H^{(e)} + H^{(p)}$  seems to be a sensitive issue for further investigation.

Another effect enclosed in Hamiltonian (26) is concerned with the anomalous magnetic moment of the electron. A Lorentz-violating study on this issue was developed in Refs. [22]. The electron magnetic moment is  $\mu = -\mu\sigma$ , with  $\mu = e/2m_e$ , and  $g = 2$  the gyromagnetic

factor. The anomalous magnetic moment of the electron is given by  $g = 2(1 + a)$ , with  $a = \alpha/2\pi + \dots = 0.00115965218279$  representing the deviation (value in the year 2008) in relation to the usual case. In this case, the magnetic interaction is  $\mathbf{H}' = \mu(1 + a)\sigma \cdot \mathbf{B}$ . In accordance with very precise measurements and QED calculations [23], precision corrections to this factor are now evaluated at the level of 1 part in  $10^{11}$ , that is,  $\Delta a \leq 3 \times 10^{-11}$ . In our case, the Hamiltonian (26) provides tree-level LV contributions to the usual  $g = 2$  gyromagnetic factor, which can not be larger than  $a$ . The total magnetic interaction in Eq. (26) is

$$\frac{e}{2m_e} (\sigma \cdot \mathbf{B}) + \lambda^{(e)} (\sigma \cdot \tilde{\mathbb{B}}). \quad (39)$$

For the magnetic field along the z-axis,  $\mathbf{B} = B_0 \hat{z}$ , and a spin-polarized configuration in the z-axis, this interaction assumes the form

$$\mu \left[ 1 + \frac{2m_e}{e} \lambda^{(e)} (\kappa_{HB})_{33} \right] (\sigma_z B_0), \quad (40)$$

with  $\frac{2m_e}{e} \lambda^{(e)} (\kappa_{HB})_{33}$  representing the tree-level LV correction that should be smaller than  $a$ . Under such consideration, we obtain the following upper bound:

$$\left| \lambda^{(e)} (\kappa_{HB})_{33} \right| \leq 9.7 \times 10^{-11} (eV)^{-1}, \quad (41)$$

where we have used  $m_e = 5.11 \times 10^5 eV$ ,  $e = \sqrt{1/137}$ .

Finally, we should claim the nonrelativistic Hamiltonian (26) possess a Rashba-like coupling term,  $\frac{\lambda^{(e)}}{m_e} \mathbb{E} \cdot (\sigma \times \mathbf{p})$ . Indeed, the Rashba spin-orbit interaction (RSOI), given by  $H_R = \beta_R (\sigma_x p_y - \sigma_y p_x)$ , has been studied in many works [24]. In Refs. [25], the usual Rashba coupling term is examined in connection with quantum transport properties of ring systems, where Aharonov-Casher effect leads to well defined conductance oscillations. A recent work has also argued that some terms of the fermion sector of the SME induces a Rashba-like coupling term [26]. In accordance with Refs. [24], the Rashba constant for a typical mesoscopic system is  $\beta_R \simeq 10^{-12} (eV \cdot m) = 5 \times 10^{-6}$ . For a typical electric field ( $E \simeq 10^7 \text{ Volt/m} = 23.1 (eV)^2$ ), the factor  $E/m_e$  is approximately  $4 \times 10^{-5} eV$ . The imposition of the condition  $\frac{\lambda^{(e)}}{m_e} |\mathbb{E} \cdot (\sigma \times \mathbf{p})| < \beta_R$  leads to  $\left| \lambda^{(e)} (\kappa_{DE})_{3j} \right| < 0.1$ , revealing that the Rashba coupling phenomenology is not a good route to constrain this nonminimal coupling.

## V. CONCLUSIONS

We have devised a new CPT-even and Lorentz-violating nonminimal coupling between fermionic and gauge fields. This dimension-five nonminimal coupling involves the dimensionless tensor  $(K_F)_{\mu\nu\alpha\beta}$  which composes the CPT-even and Lorentz-violating electrodynamics of the SME. It was considered in the context of



the Dirac equation and the nonrelativistic limit was assessed and carried out. The resulting nonrelativistic Hamiltonian possess new interesting contributions able to yielding a new effect (a kind of electric Zeeman-like effect), corrections to the anomalous magnetic moment, and a Rashba-like coupling term. The electric Zeeman-like effect may lead to upper bounds as good as  $|\lambda^{(e)}(\kappa_{HE})_{3j}| < 8 \times 10^{-17} (eV)^{-1}$ , while the corrections on the magnetic moment yields an upper bound as tight as 1 part in  $10^{10}$ . It is important to mention that the bounds here found should not be confused with the upper bounds on the  $(K_F)$ -CPT-even components already known in the literature, once in the present case the constraint is on the magnitude of the CPT-even parameters as nonminimally coupled. Note that adopting distinct configurations of the electric and magnetic fields (not along the  $z$ -axis), similar upper bounds can be imposed on other coefficients of the matrices  $(\kappa_{HE})$  and  $(\kappa_{HB})$ .

Concerning this dimension-five nonminimal coupling a promising investigation is related with the radiative corrections stemming from the photon one-loop vacuum polarization. Carrying out such radiative contributions, we observe that a dimension-four CPT-even term,  $(\lambda^{(e)}m_e)(K_F)_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ , is generated. Dimension-six operators are also generated at second order in  $\lambda^{(e)}K_F$ . Since the dimension-4 operator can be generated by radiative corrections, the existing bounds [9, 10] on the CPT-even  $(K_F)_{\mu\nu\alpha\beta}$  can be used to achieve even better bounds on the magnitude of the quantity  $\lambda^{(e)}(K_F)_{\mu\nu\alpha\beta}$ .

The fact that this term has as coefficient  $(\lambda^{(e)}m_e)$  allows to attain better bounds on  $\lambda^{(e)}K_F$  by the factor  $1/m_e \sim 10^{-5}$  in comparison with the bounds on  $K_F$ . For instance, typical bounds on the nonbirefringent coefficients,  $|\lambda^{(e)}K_F| < 10^{-18} (eV)^{-1}$ , imply upper bounds as tight as  $|\lambda^{(e)}K_F| < 10^{-23} (eV)^{-1}$  on the nonminimal coupling. The detailed analysis of this issue is now under consideration.

Another interesting perspective is concerned with a complete investigation of the corrections on the Hydrogen spectrum implied by the Hamiltonian (25) and  $H = H^{(e)} + H^{(p)}$ . Such analysis should be carefully carried out for all the terms involving  $E, B, \vec{E}, \vec{B}$ , focusing on the ones that could yield stringer upper bounds on the LV parameters, and having as counterpart the procedures of Refs. [17], [27].

Finally, this new coupling may be examined in several distinct respects, including applications in the ultrarelativistic regime. Very recently, a study involving an electron-positron scattering in a QED framework endowed with this nonminimal coupling was successfully performed yielding upper bounds as tight as  $\lambda^{(e)}(K_F) < 10^{-12} (eV)^{-1}$  [28].

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